Harder Problems

- 1. (a) Solve the simultaneous equations equations $y = 2x^2 + x + 1$ y + 17 = 13x
 - (b) What does your answer say about the geometric relationship between the two in the xy-plane?
- 2. Solve $\frac{5}{x-1} + \frac{9}{x+1} = 8$.
- 3. Make x the subject of $\frac{ax-b}{cy+dx} = e$.
- 4. Find the range of the function $f(x) = 7 + 4x x^2$ if the domain is all the real numbers. [Hint: Sketch]
- 5. Given $f(x) = \frac{ax}{b-x}$ find
 - (a) $f^{-1}(x)$. (b) ff(x). $f^{-1}(x) = \frac{bx}{a+x}$
 - (c) fff(x).
- 6. Given $g(x) = \frac{3x-a}{7-kx}$ find
 - (a) $g^{-1}(x)$. $g^{-1}(x) = \frac{7x+a}{3+kx}$
 - (b) gg(x). $gg(x) = \frac{9x 10a + akx}{49 10kx + ak}$ (c) ggg(x). $\frac{27x 79a + 13akx a^2k}{242 70k + 137k + a^2k}$
- 7. The normal to $y = 3x^2 x + 2$ has gradient 4. Find where on the curve the normal exists.
- 8. Find the equation of the tangent to $y = x^2 + 2x 1$ when x = p in the form ax + by + c = 0. $(2p + 2)x + y + 1 + 2p^2 = 0$
- 10. The tangent to $y=x^2$ when x=p crosses the x-axis at P and the y-axis at Q. If the origin is O, find the area of triangle OPQ.
- 11. Differentiate the following with respect to x:
 - (a) (x+2)(x-1)(x+3). (b) (x+a)(x-a)(x+b). $3x^2 + 8x + 1$ $3x^2 + 2bx - a^2$
 - (c) $3\sqrt{x} \frac{1}{2x}$. (d) $\frac{3x^3 - 7x^2 + 2x - 5}{x^2}$. $3 - \frac{2}{x^2} + \frac{10}{x^3}$
 - (e) $\frac{(\sqrt{x}+x^2)(x-3)}{\sqrt{x}}$. $1+\frac{5}{2}x^{\frac{3}{2}}-\frac{9}{2}\sqrt{x}$
- 12. Find the point of intersection of the tangent to $y = x^3 3x + 1$ when x = 1 and the normal when x = -1.
- 13. Find the point of intersection of the tangent to y = (2x 1)(x + 3) when x = 1 and the normal when x = 2.

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